
第9日目のまとめ

線形写像

山本健一

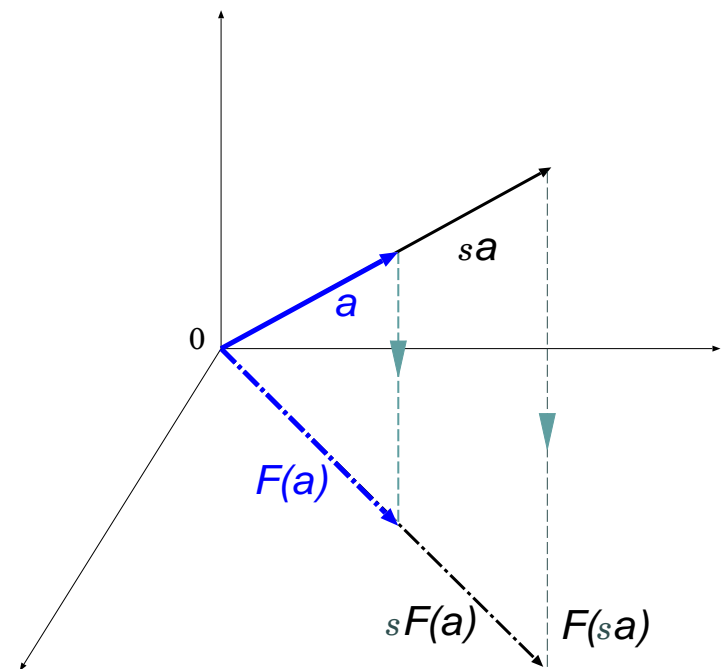
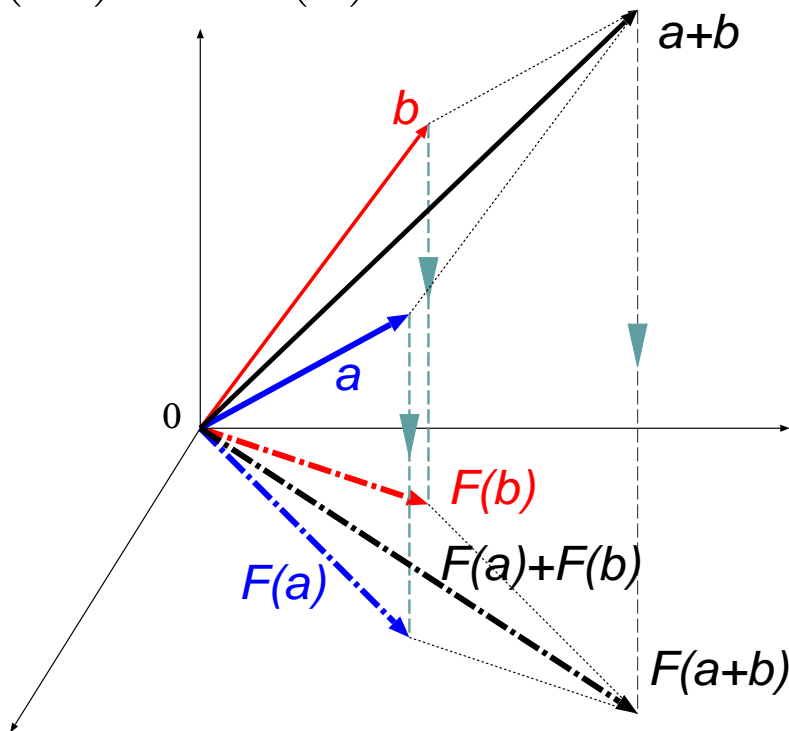
<http://mag.eee.u-ryukyu.ac.jp/linalg/>

線形写像の定義

次の2つの条件を満たす写像 $F : V \rightarrow W$

● $F(x_1 + x_2) = F(x_1) + F(x_2)$

● $F(sx) = sF(x)$

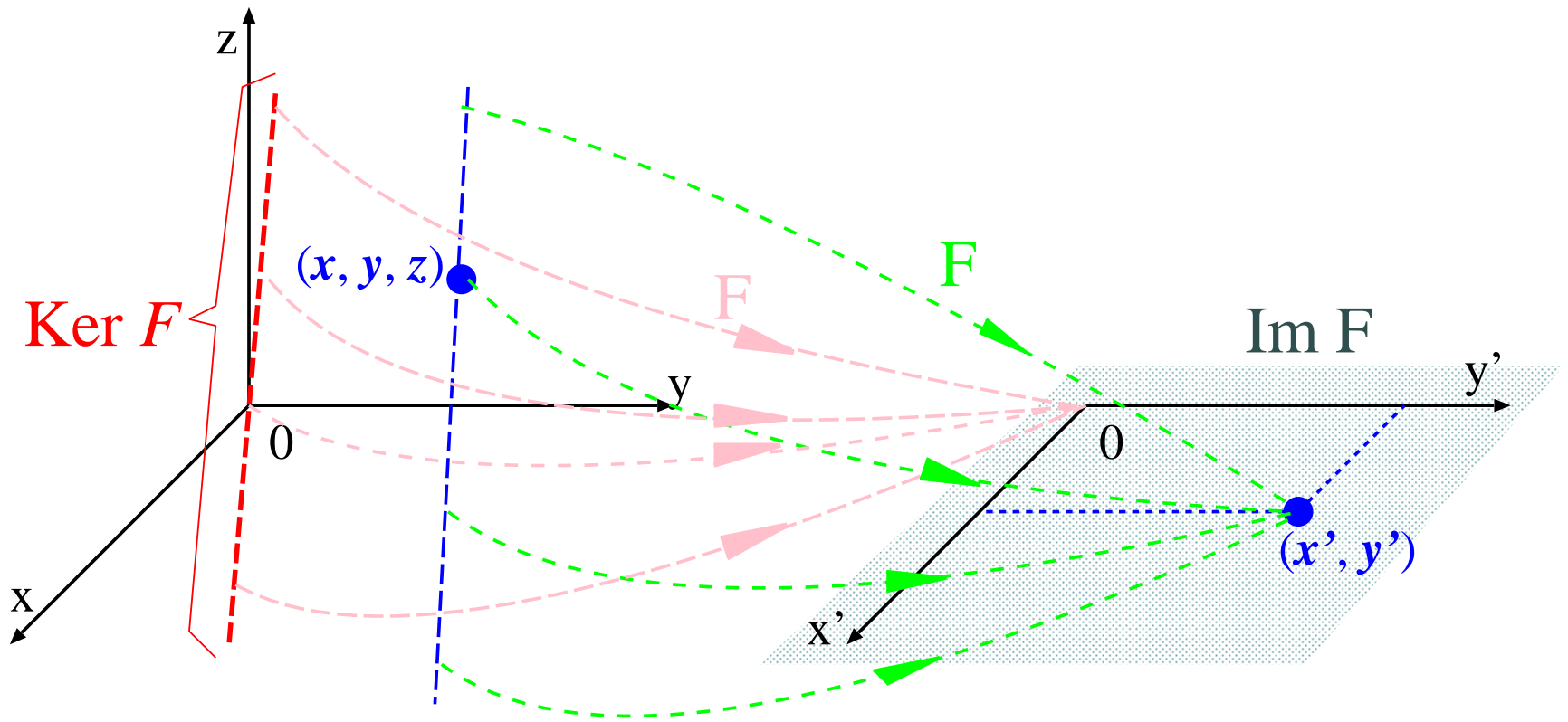


V から V への線形写像 \implies **線形変換**

像と核

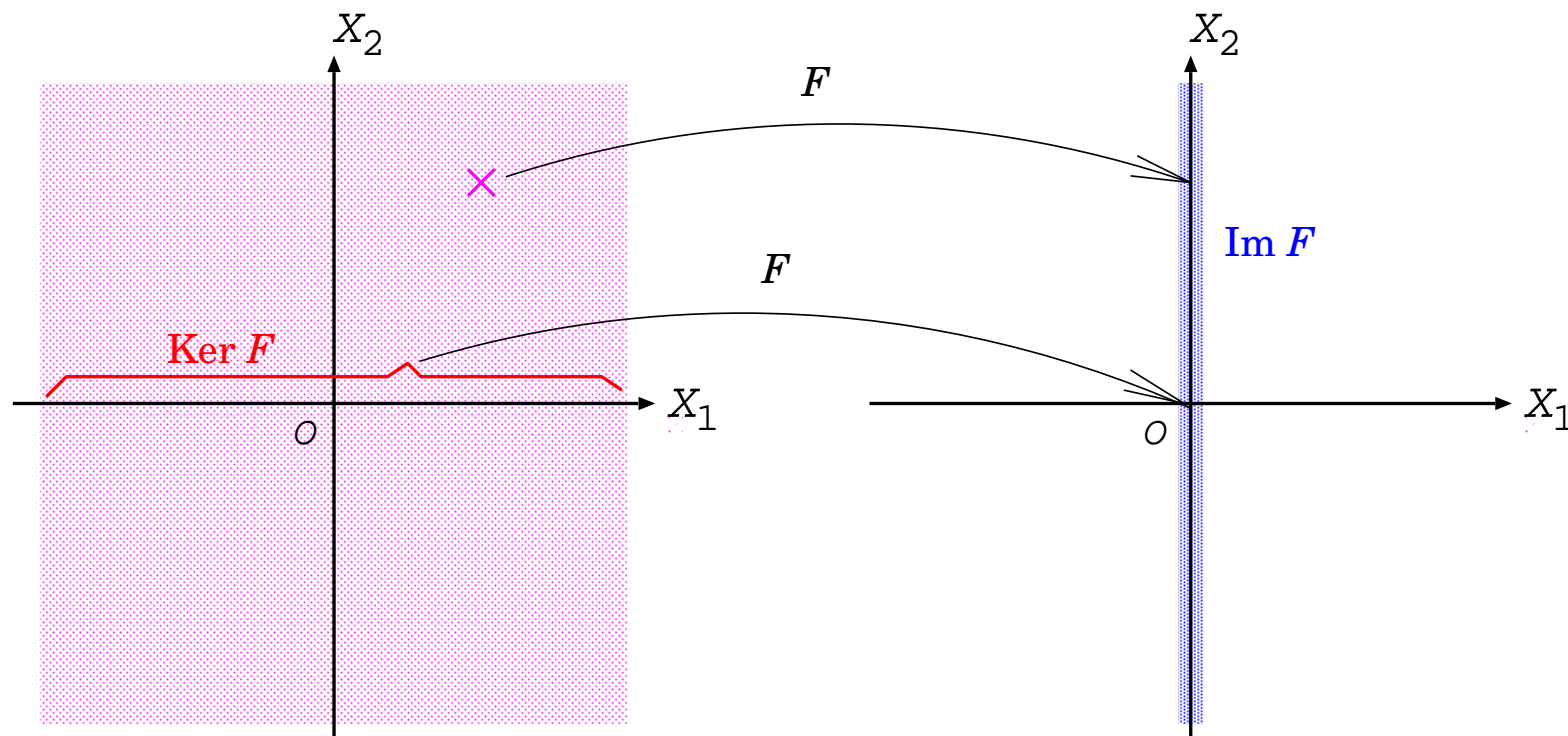
線形写像 $F : V \rightarrow W$ に対して

- $\text{Im}F = \{F(\boldsymbol{x}) \mid \boldsymbol{x} \in V\}$ を F の像 (image)
- $\text{Ker}F = \{\boldsymbol{x} \mid F(\boldsymbol{x}) = \mathbf{0}\}$ を F の核 (kernel)



線形写像の例(1)

$$\mathbf{y} = F(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$$

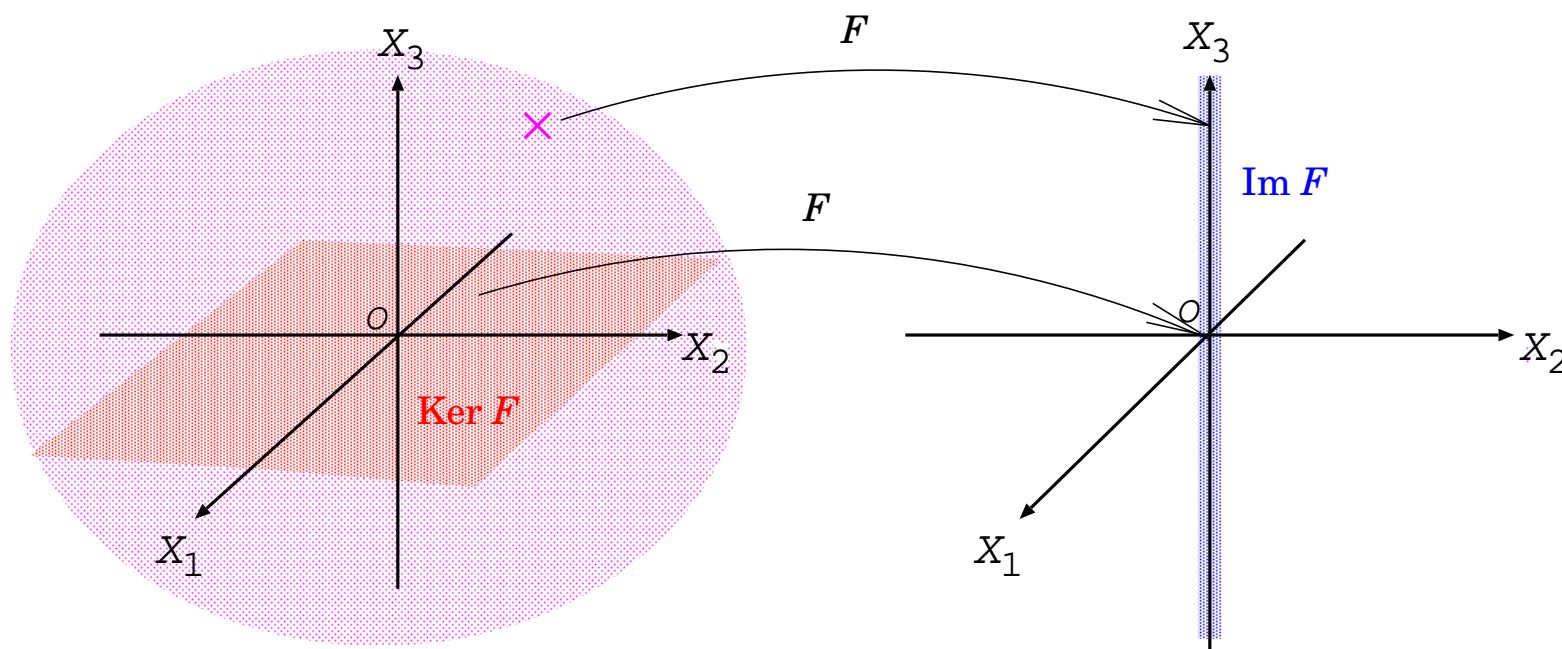


$$\dim V = 2, \quad \dim(\text{Ker } F) = 1, \quad \dim(\text{Im } F) = 1$$

$$\dim V - \dim(\text{Ker } F) = \dim(\text{Im } F)$$

線形写像の例(2)

$$y = F(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix}$$

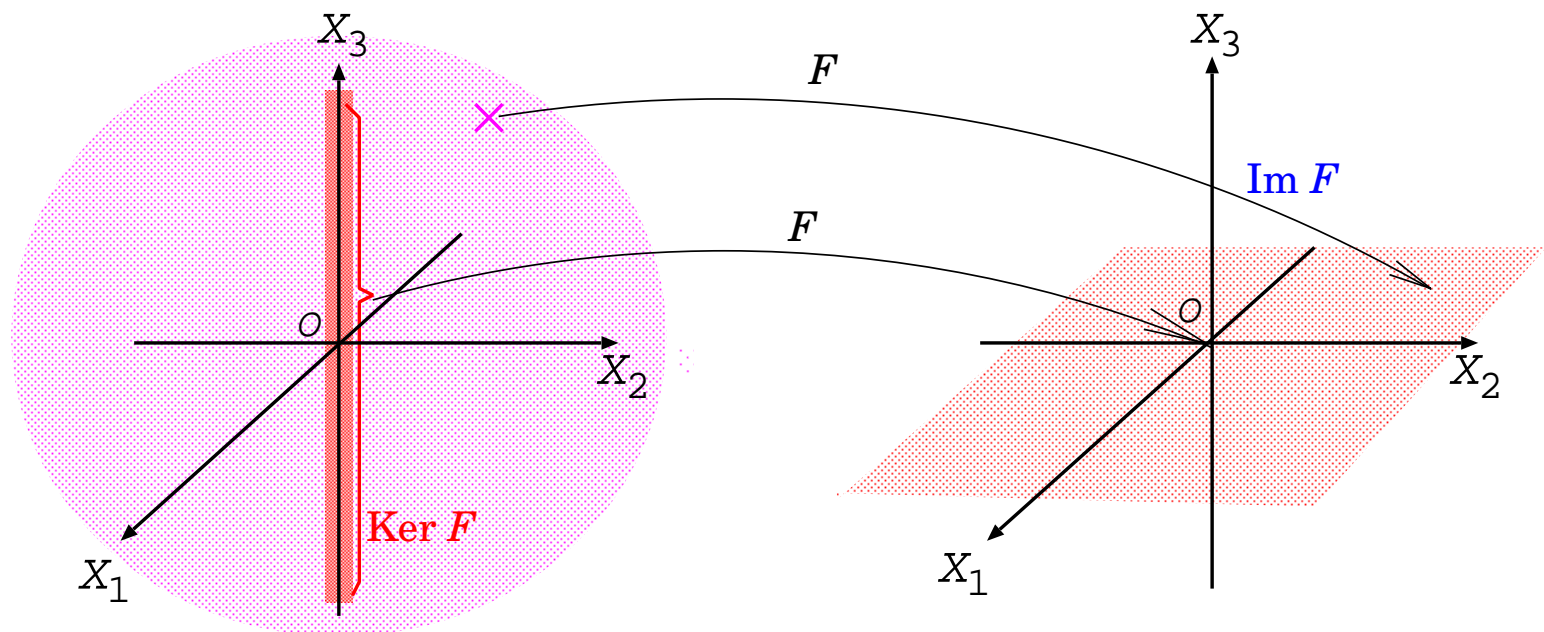


$$\dim V = 3, \quad \dim(\text{Ker } F) = 2, \quad \dim(\text{Im } F) = 1$$

$$\dim V - \dim(\text{Ker } F) = \dim(\text{Im } F)$$

線形写像の例(3)

$$\mathbf{y} = F(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$$



$$\dim V = 3, \quad \dim(\text{Ker } F) = 1, \quad \dim(\text{Im } F) = 2$$

$$\dim V - \dim(\text{Ker } F) = \dim(\text{Im } F)$$

連立一次方程式

$$\bullet \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{cases}$$

$$\bullet Ax = 0, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

● 「方程式 $Ax = 0$ の解全体 (解空間)」を
「行列 A の零空間」ともいう。

● $Ax = 0$ を満たす x は, A による線形写像の核。